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Cadastral cartography: from local “small systems” to global GNSS ones

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Abstract The adoption of satellite measurement techniques also for survey operations and cadastral updating, introduces the issue of transformation between the old reference systems, in which the mapping was generated towards modern reference systems. The problem, in the cadastral case, is further complicated by various factors of technical, scientific and even historical origin. These are inherent to the methods used to produce the reference system that form the frame of cadastral maps. A geodetic procedure has already been presented in this journal which allows the transformation between cadastral and global reference systems, based on knowledge of the cadastral origin and some points of double coordinates. Here, a new and more complex procedure is introduced. It is to be applied to a set of reference systems of small extension called “small origins”, in which the coordinates of a number of points, sufficient to apply the previous transformation method, are not available.

The new method is named "Cadastral Aerial Triangulation" (CAT), due to its analogies with photogrammetric aerial triangulation: following this analogy, we can say that it requires a limited number of "control" points for each "block", formed by a set of cadastral origins. In addition to that, it is necessary to have some common points for each cadastral origin, with known coordinates in both local reference systems. In general, these points are located in the perimeter zones of each local reference system, and are similarly the "tie points" in aerial triangulation

In this paper, after the introduction of the method, the results concerning a large Province of the Piedmont region, are illustrated critically. This allows us to evaluate the accuracy of the original cadastral networks, used for the construction of the original cadastral map. The method was then extended to the entire Piedmont Region with the same success.

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1 | INTRODUCTION

Cadastral cartography is an important instrument for territory management, for property assessment and boundary regulation. Even if it refers mainly to property questions, it is generally one of the few large-scale maps of the country. Conceived and established in certain areas as far back as a couple of centuries ago, it has been periodically updated to follow the evolution of the territory and its properties, by using the most modern survey techniques up to and including GNSS. In past times, human activities were territorially more limited than they are nowadays, and even local reference systems with small extensions of only a few kilometers, were adequate for purpose. Their external incoherence rapidly necessitated increased reference system extensions in order to reduce the number of these. In the process, non-homogeneous situations were created in Italy, with coexistence of origins of both small and large size, still inconsistent among themselves and with global reference systems in which GNSS surveys operate. This GNSS technique is currently also use for cadastral surveying operations.

In order to maintain consistency with technical maps and satellite positioning systems, it is very important to first unify the local reference systems and, subsequently, to make them compatible with the global system. After performing these operations, simple and fast RTK measurements are sufficient, it would therefore be possible to perform/carry out surveys for division or reconfinement operations, directly on the ground in real time, with precision of a few centimeters and, above all, also in areas between two or more Municipalities.

However, this topic also involves the broader scenario concerning the metric accuracy of cadastral maps and even how this accuracy may have been maintained over time, through a number of updating and digitizing phases of paper sheets.

This aspect will not be treated here, our specific focus being the question of the unification of the reference systems in which these maps were created. It is easy to understand how this does not depend on the subsequent detailed surveying operations with which they were constructed, nor on the map building works and on the degradation and updating of the maps produced over time. On the other hand, it will be necessary to go back to the problem of external congruence among the various reference systems.

The need for this transformation is also linked to the evolution of national and international reference systems. In addition, in Italy, the INSPIRE directive, implemented by the legislative Decree of 10 November 2011 must be taken into account. It provides for the adoption of the new national geodetic system based on the RDN (National Dynamic Network) and the cartographic representation of Gauss in the international UTM standardization.

Assuming that the deformations of the territory from the origin to today are negligible, which is obviously not true, and that measurement errors are considered accidental, the exchange between reference systems is performed by comparing the coordinates of the same points on two or more maps, where these "double points" are recognized in the two systems.

The transformation model must be compatible with the cartographic projection from which the coordinates of the maps that have common points originate. However, in order to evaluate the tolerances of the proposed method, it is necessary to know the survey methodologies that were adopted at the time.

The article proposes a rigorous geodetic procedure, which can also be used when double points are not sufficient to frame independently each single local reference system in the global system. This method can also be applied to large territories (provinces, regions or nations), taking into account the various reference systems and the different cartographic deformations induced by the multiple types of cartographic representations.

2 | GEODETIC AND CARTOGRAPHIC CONSIDERATIONS

Reference system (datum) transformation in the geodesy is usually performed by estimating the transformation parameters according to a given model: generally the 3D roto-translation with scale variation or its linearization represented by the Molodensky transformation. In both cases, it is necessary to compare coordinates of known points in the two reference systems.

A large part of the cadastral cartography has used the Bessel ellipsoid oriented in Genova with Cassini Soldner cartographic projection. This is particularly true for the North of Italy, which was created first, however, due to the limited size of the small origins, the vertical deviation has been considered null. For this reason instead of talking about "many cartographic systems", the expression "many reference systems" should be preferred.

However, it is customary to say that, for most cadastral cartography, the datum is the ellipsoid of Bessel of 1841 oriented to Genova, and the cartographic projection for each origin is that of Cassini Soldner, without further specification.

We will also consider this hypothesis as being correct because it leads back to what was used during the genesis of the maps. We hypothesize the datum to be unique for the various origins and that, with particular procedures, the various cadastral coordinates might be used for transformation to pass from the old to the new reference system and from the old to the new cartographic system.

The approach followed starts with the coordinates of the two different cartographic projections related to two reference systems.

It is known that the Gauss and Cassini Soldner cartographic representations have different linear and angular deformations. The basic approach of a conformal transformation among coordinates is not, therefore, rigorous and would lead to discrepancies incompatible with the cadastral precisions even in origins of limited extension and, for the merging of lots of a small reference system.

The rigorous procedure is explained in detail in (Cina, 2008), and therefore, only the fundamental steps are recalled here, since they are crucial for a clear understanding of the next phase, allowing many reference systems to be gathered together. The problem is solved in the following two phases:

1) transformation from the Cassini Soldner coordinates to the Gauss coordinates, expressed in the cadastral datum;
2) transformation from the cadastral datum to another datum starting from the Gauss coordinates in both systems.

The method is mainly based on the fact that, in the change of datum between the two reference systems already mentioned, the geodesic arch "s" and its azimuth "α" (Figure 1) are practically invariant for practical purposes: This in the hypothesis that the values of s and α sare however definite quantities on the surface of the ellipsoid itself (Cina, 2008; Surace, 1991).

It is possible to perform the transport of the cartographic coordinates on the Gauss map from a point O of coordinates E_O, N_O, to a point P (Figure 1), just by using the length s "and azimuth α of the transform of the geodesic OP with the:

$$
\begin{aligned}
E_P &= E_O + s''_{OP} \sin \vartheta_{OP} \\
N_P &= N_O + s''_{OP} \cos \vartheta_{OP}
\end{aligned}
\qquad (1)
$$

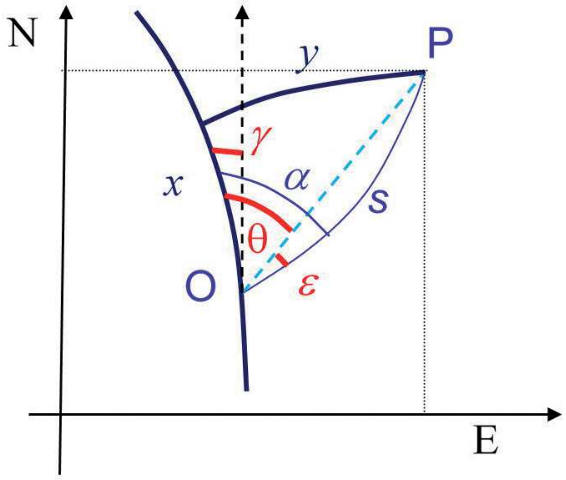


Figure 1 Geodesics in the plan of Gauss and Cassini Soldner - common points

With s" and ϑ representing respectively the length and the angle of direction of the chord underlying of the transform of geodetic line. These values can be obtained first from the X, Y cadastral coordinates via the Soldner formulas (Inghilleri, 1970). On Cassini Soldner map the length and azimuth of the geodesic are expressed by:

$$s = \sqrt{(Y + \varepsilon X)^2 + (X + 2\varepsilon Y)^2} \qquad \alpha = atn\frac{Y + \varepsilon X}{X + 2\varepsilon Y} \tag{2}$$

$3\varepsilon = \frac{XY}{2\rho N}$ with spherical excess and ρ, N main radiuses of curvature of ellipsoid. By recalling the conformity characteristics of the Gaussian cartographic representation, on it, the angle of direction of the chord underlying the geodesic transformation can be obtained from:

$$\vartheta_{OP} = \alpha_{OP} + \gamma_O - \varepsilon_{OP} + \delta_{OP} \tag{3}$$

where γ it is the convergence of the meridian transform, ε is the angular correction to the chord and δ is the angular deformation of the Cassini Soldner representation.

As regards the length of the OP chord, let us start from the length of the geodesic line s on the Cassini Soldner representation plane. It must be "re-projected" on the ellipsoid with the linear deformation module of the Cassini Soldner representation (m_{CS}) and displayed on the Gauss plane with the related linear deformation module (m_{Gauss}). Let us name s' this reduced size on the Gaussian cartographic representation:

$$s' = s\frac{m_{Gauss}}{m_{CS}} \tag{4}$$

Finally, in the change of *datum* between locally oriented ellipsoids, it is assumed that the reduction of the distance to the reference surface has taken place starting from the orthometric height *H* instead of the ellipsoid one *h*. The operation is legitimate given that, on the local ellipsoids, the undulations of the geoid *N=h-H* are of the order/size of a few meters: if neglected, the reduction from geoid to ellipsoid leads to a residual error of the order of 10-6, , minor for practical goals.

If we consider geocentric systems such as WGS84, this reduction must be considered. In this system the values of geoid undulation can also be of several tens of meters (about 50 m in the case of Piedmont). Therefore, a further reduction of distance is considered:

$$s'' = s' \left(1 - \frac{h-H}{\sqrt{\rho N}}\right) \tag{5}$$

This length *s”* is the value to be used in (1). The length of the geodesic transform can be confused with the one of the chords without loss of precision (10^{-7} per 1000 km).

Returning to the coordinates of the Gaussian cartographic representation and taking advantage of the concept, expounded/explained previously, that the values of s remain almost unchanged at the change of ellipsoid and azimuth α are modified by a common constant, the Gaussian coordinates in the cadastral datum will be, after such reductions, only rotated and translated compared to the global reference ellipsoid. In addition to that, if the coordinates TX and TY of the origins in the arrival datum are known, it will be theoretically necessary to estimate only the rotation parameters according to the conformal transformation

$$\begin{pmatrix} E \\ N \end{pmatrix} = \begin{pmatrix} T_X \\ T_Y \end{pmatrix} + \lambda \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} \tag{6}$$

Actually, it would be correct to estimate not only the unknown rotation resulting from the different orientation between the two ellipsoids, but also the scale factor that, after the already applied reductions, can still derive both from the inevitable average distortions of the geodetic networks that materialized the two *data* and the dimensions of the relative reference ellipsoids.
On such a basis, and according to the scheme outlined above, a software procedure called CS2UTM was developed at the *Politecnico di Torino* to automate the transformation starting from the knowledge of the origin in the *datum* of “arrival” and a few points of double coordinates for the estimate of β and λ.

Even in the case of uncertain or lost origin coordinates, the origin position can be estimated, with the aforementioned procedure, starting from some points, theoretically at least two of double coordinates, known by monograph, or with the help of points still recognizable in the map and detectable with modern geodetic measurements.
In their absence, if the origin of reference system is traceable on the ground, it can itself be detected. Given that, in general, this origin is hardly usable with GNSS, intersection operations and mixed GNSS and classic topographic techniques will be required.
If such information, however, were not available, in order to apply the method, it would be necessary to adopt the procedure under examination in this study.

3 | CATASTAL AERIAL TRIANGULATION (CAT)

The previous CS2UTM procedure requires the origin coordinates in the arrival *datum* to be known, as well as some double coordinate points (generally trigonometric markers). This requirement cannot always be obtained, especially in small-scale origins that may not have trigonometric markers of double coordinates within them.

The new procedure makes it possible to perform together the transformation of a series of origins towards a single reference system with the use of the historical data of the cadastral networks, which have constituted the materialization of each local origin, provided they are common to two or more reference systems of adjacent areas. It is necessary to have a limited number of double coordinatepoints, not necessarily known in each origin.

Let us then look at the following similarities with what happens in photogrammetry for an independent model triangulation, limiting ourselves to the planimetric unknowns. (Krauss, 1994):

1) the cadastral cartographic system is identical to the local system of each photogrammetric model;
2) many network points, employed in the formation of the cadastral network, have been used with some multiplicity in more than one origin. In each of these, they assume different coordinates. These vertices between several adjacent origins can be assumed as the tie points in the aerial triangulation;
3) the points of the cadastral network, whose coordinates are also known in the global reference system (generally are also points of the IGM trigonometric network), are analogous to the ground control points.

We will better understand the analogy through the example of Figure 2, where we can observe:

- three local cadastral systems with origins O1 O2 and O3, and different orientations among them;
- network points common to multiple sources. They are going to be numbered in apex: 2' and 2", 4" and 4"', 6' 6" and 6"', 7' e 7"
- Let us now observe the three points of support 1, 3 and, 5 in Figure 3: they would not be sufficient if we wanted to orientate the local systems individually. They are, however, sufficient and redundant to get the origins in the global system and in the Gauss projection (coordinates East, North).

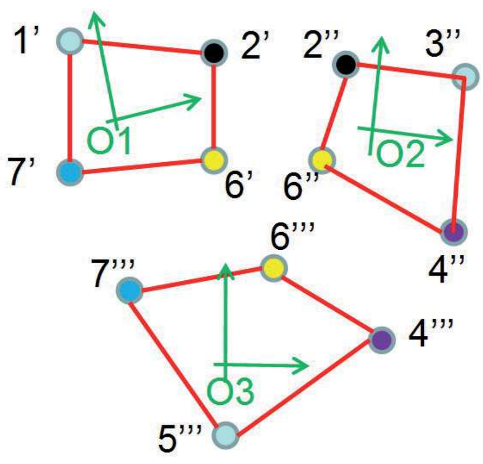


Figure 2 Cadastral origins in local systems with common points - *Source: author's elaboration*

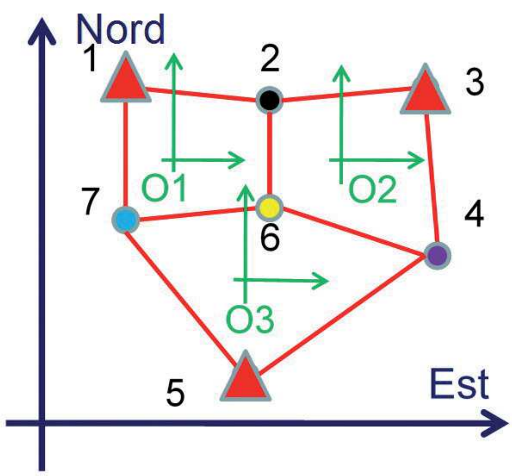


Figure 3 Cadastral origins in the global system, "tie" and "control" points - *Source:Source: author's elaboration*

Let us study the planimetric transformation, starting from the relationships of conformity among global and local reference systems, after having transformed the coordinates from the Cassini Soldner to the Gauss projections.
As mentioned, the hypothesis consists in starting from pairs of coordinates, deriving from different projections and reference systems but transformed in the same cartographic Gauss projection.
The analogy of triangulation, but also the model of linear transformation expressed by (7) would not be, with two different systems, therefore, correct.
The problem has been solved using recursive approximations: the hypothesis of conformal transformation among non-coherent cartographic systems leads to a first approximate estimate of the coordinates Tx and Ty of the cadastral origins in the Gaussian projection, even using temporary origins, starting with low accuracy of orientation and scale factor, with the illustrated geodetic procedure.
After the recursive procedure, when the pairs of coordinates are both in the Gaussian projection, we can write the equations for each “tie point”:

$$\begin{aligned} &\overline{T}_X \quad + \overline{a}X + \overline{b}\,Y - \overline{E} = v_X \\ &\quad \overline{T}_Y - \overline{b}X + \overline{a}\,Y - \overline{N} = v_Y \end{aligned} \qquad (7)$$

While, for every “ground control point” we can write:

$$\begin{aligned} &\overline{T}_X \quad + \overline{a}X + \overline{b}\,Y = E + v_X \\ &\quad \overline{T}_Y - \overline{b}X + \overline{a}\,Y = N + v_Y \end{aligned} \qquad (8)$$

In order to get to the redundant system Ax – l = v, o be solved via Ordinary Least Squares.
The (7) and (8) do not differ in form but in the number of unknowns (marked terms): coordinates E, N in the global system are actually known only for control points. Parameters of transformation are common in equations involving points in the same origin. The balance equations-unknowns is, therefore:

Unknown parameters:
- 4 unknown parameters (T_X, T_Y, a, b) for each local system;
- 2 coordinates (E, N) unknown for each tie point among local origins (equations 7);

Equations:
- 2 for each point of known coordinates E, N note: (equations 8);
- 2 constraint equations for each tie point *i* common to the two origins:

$$E_i^{'} = E_i^{''} \qquad\qquad N_i^{'} = N_i^{''}$$

If we apply the balance to the proposed case in Figures 2 and 3, we will get the following unknown parameters:
- local systems: 4 per origin * 3 origins = 1
- coordinates of the tie points 2, 4, 6 and 7: 2 per 4 points = 8

for a total of 20 unknowns.

There will be two equations for each tie and control point for each origin. In this example, we are going to have for the various origins:

- 2 equations for each point of support (1, 3 and 5) = 6 equations;
- 2 for tie points of origin O_1: 3 points (2', 6', 7') * 2 coordinates = 6 equations;
- 2 for tie points of origin O_2: 3 points (2", 4", 6") * 2 coordinates = 6 equations;
- 2 for tie points of origin O_3: 3 points (4''', 6''', 7''') * 2 coordinates = 6 equations.

PFor a total of 24 equations. The analyzed system is just an example, and has low redundancy: 24-20=4, but it already permits us to estimate the unknown parameters using ordinary least squares procedure $\hat{x}$:

$$\hat{x} = \left(A^T P A\right)^{-1} A^T P l = N^{-1} T_n \tag{9}$$

Remaining on the example of Figures 2 and 3, we explicit the design matrix A(24x20) for greater clarity. The vector of parameters x and the one of known terms are:

$$\left[\begin{array}{c:c:c:cccc}
 & & & 0 & 0 & 0 & 0 \\
A_1 & 0 & 0 & I & 0 & 0 & 0 \\
 & & & 0 & 0 & I & 0 \\
 & & & 0 & 0 & 0 & I \\
\hdashline
 & & & 0 & 0 & 0 & 0 \\
0 & A_2 & 0 & I & 0 & 0 & 0 \\
 & & & 0 & I & 0 & 0 \\
 & & & 0 & 0 & I & 0 \\
\hdashline
 & & & 0 & 0 & 0 & 0 \\
0 & 0 & A_3 & 0 & I & 0 & 0 \\
 & & & 0 & 0 & I & 0 \\
 & & & 0 & 0 & 0 & I
\end{array}\right]
\begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ EN_2 \\ EN_4 \\ EN_6 \\ EN_7 \end{pmatrix}
-
\begin{pmatrix} EN_1 \\ 0 \\ 0 \\ 0 \\ EN_3 \\ 0 \\ 0 \\ 0 \\ EN_5 \\ 0 \\ 0 \\ 0 \end{pmatrix}
= \left(v^{(24x1)}\right) \tag{10}$$

Where submatrices A, P, I e EN are expressed by:

$$A_1^{(8x4)} = \begin{bmatrix}
1 & 0 & X_1' & Y_1' \\
0 & 1 & Y_1' & -X_1' \\
1 & 0 & X_2' & Y_2' \\
0 & 1 & Y_2' & -X_2' \\
1 & 0 & X_6' & Y_6' \\
0 & 1 & Y_6' & -X_6' \\
1 & 0 & X_7' & Y_7' \\
0 & 1 & Y_7' & -X_7'
\end{bmatrix}
\quad
A_2^{(8x4)} = \begin{bmatrix}
1 & 0 & X_3' & Y_3' \\
0 & 1 & Y_3' & -X_3' \\
1 & 0 & X_2' & Y_2' \\
0 & 1 & Y_2' & -X_2' \\
1 & 0 & X_4' & Y_4' \\
0 & 1 & Y_4' & -X_4' \\
1 & 0 & X_6' & Y_6' \\
0 & 1 & Y_6' & -X_6'
\end{bmatrix}
\quad
A_3^{(8x4)} = \begin{bmatrix}
1 & 0 & X_5' & Y_5' \\
0 & 1 & Y_5' & -X_5' \\
1 & 0 & X_4' & Y_4' \\
0 & 1 & Y_4' & -X_4' \\
1 & 0 & X_6' & Y_6' \\
0 & 1 & Y_6' & -X_6' \\
1 & 0 & X_7' & Y_7' \\
0 & 1 & Y_7' & -X_7'
\end{bmatrix}
\quad
P_i^{(4x1)} = \begin{bmatrix} T_X \\ T_Y \\ a \\ b \end{bmatrix}_i^T
\quad
I^{(2x2)} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}
\quad
EN_i^{(2x2)} = \begin{bmatrix} E \\ N \end{bmatrix}_i \tag{11}$$

Table 1 Writing of the system Ax – l =v to ordinary least squares in the example

The matrix of the weights P, in the absence of stochastic information on the precision of the points, can be assumed to be the identity matrix. The estimate of the matrix of the variance covariance is usually performed by:

$$C_{XX} = \hat{\sigma}_0^2 N^{-1} \quad \text{with} \quad \hat{\sigma}_0^2 = \frac{\hat{v}^T P \hat{v}}{n-r} \quad \text{and} \quad \hat{v} = A\hat{x} - l \tag{12}$$

The Thomson test on the normalized residue is performed to discover the coordinates of the points that will be considered outliers:

$$w = \frac{\hat{v}}{\hat{\sigma}_v} < k_\alpha \tag{13}$$

Where α here, represents the level of significance of the test, and k_α the corresponding confidence interval. The standard deviation can be obtained from the diagonal elements of the covariance variance matrix of residuals (Cina, 2004):

$$C_{vv} = RP^{-1} = \left(AN^{-1}A^T P - I\right)P^{-1} \tag{14}$$

Let us now look at the results of this method applied to a real case with data collected in the Province of Novara in the Piedmont Region.

4 | TRANSFORMATION OF THE CADASTRAL SYSTEMS OF THE PROVINCE OF NOVARA

The procedures, here presented, have been applied throughout the Piedmont Region for the transformation from the cadastral system to ETRF2000 system. To be exact, the coordinates of the control points were expressed in the old Gauss Boaga system (Roma 40 datum) and the transformation was first performed in this system. The transformation, from the Roma 40 system to the ETRF2000 system, has been performed using the VERTO approach, starting from iso-variation "grids" of the geographical coordinates, provided by the IGM.

The issue of small origins is particularly widespread both in the province of Novara and throughout the Piedmont Region, which has over 540 of them. In particular, this concerns two of the Provinces: large parts of Novara and Alessandria. Historical data show only the coordinates of the cadastral networks (Figure 4) in local systems (Figure 6).

As an example, the province of Novara, is 1340 km2 in extension and has 88 municipalities, of which 50 are cartographically referred to small origins, and 33 referred to the large origin on the “ideal point Ω” of Vercelli, which has no materialization on the ground. Five municipalities have more than one cartographic origin. Therefore, we obtain a number of 62 “axis systems” of small extension and only one system of great extension (Figure 5).

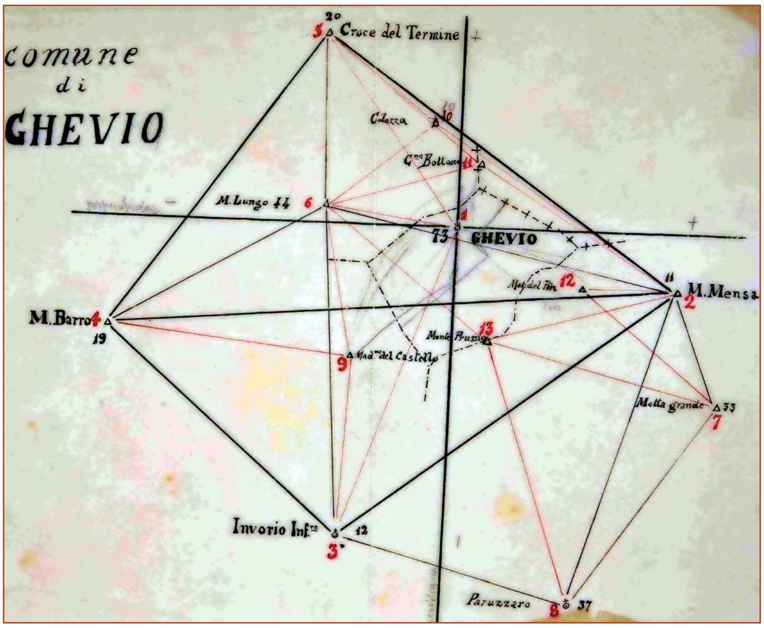


Figure 4 Reference system of a cadastral origin: networks and sub-networks
Source: author’s elaboration

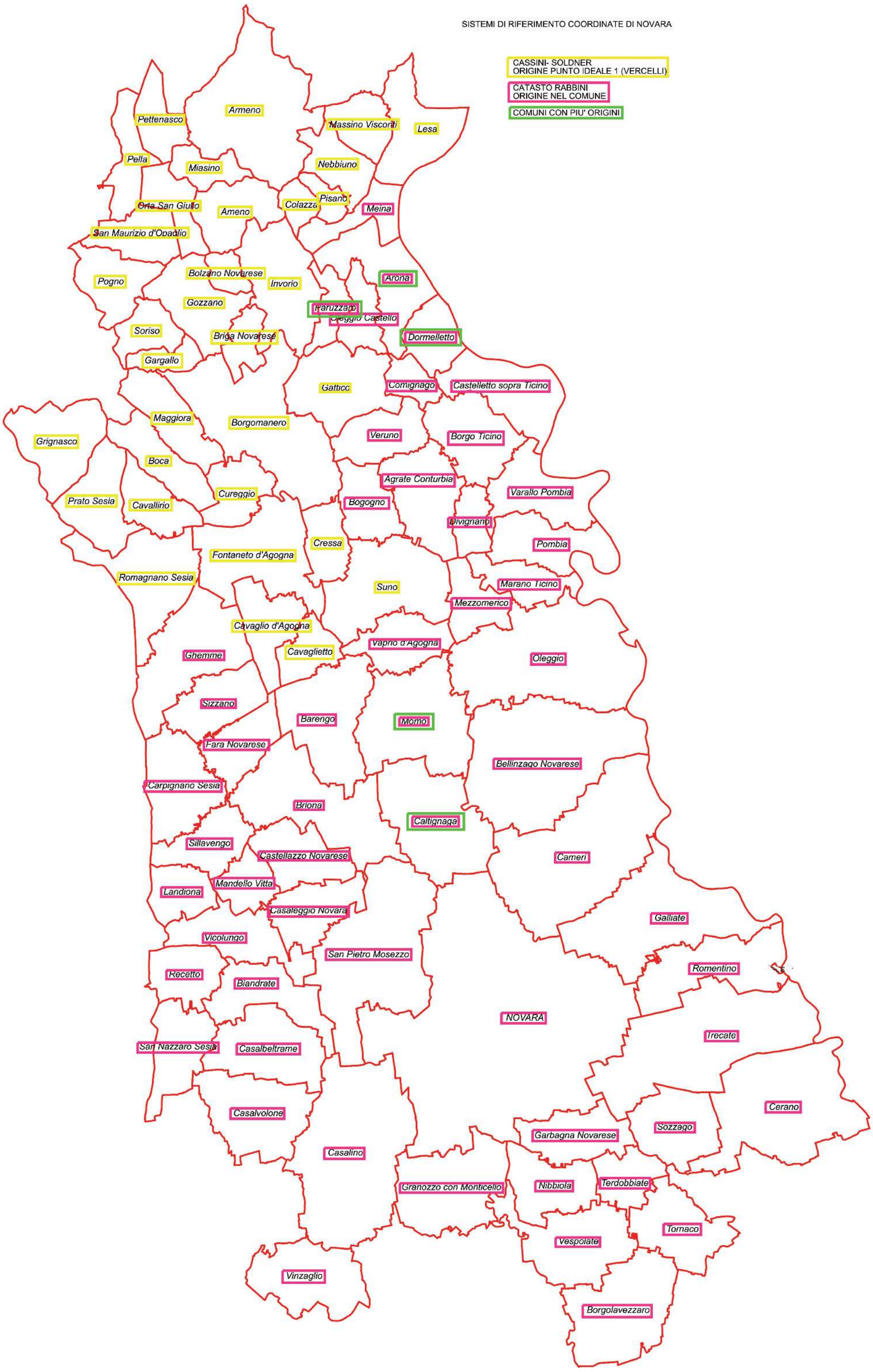
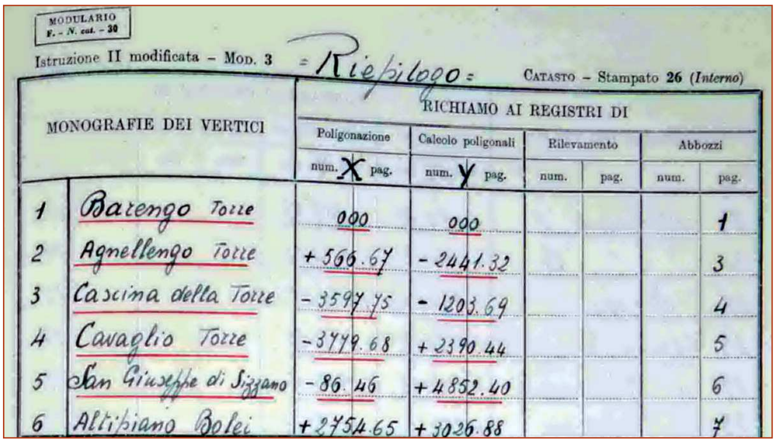


Figure 5 Origins in Novara Province - *Source: AdT Novara*
In red and green the small origins, in yellow the large origin on “ideal point” Ω



MODULARIO
F. - N. cat. - 30

Istruzione II modificata – Mod. 3 =Riepilogo= Catasto – Stampato 26 (Interno)

MONOGRAFIE DEI VERTICI		RICHIAMO AI REGISTRI DI					
		Poligonazione	Calcolo poligonali	Rilevamento		Abbozzi	
		num. X pag.	num. Y pag.	num.	pag.	num.	pag.
1	Barengo Torre	000	000				1
2	Agnellengo Torre	+ 566.67	- 2441.32				3
3	Cascina della Torre	- 3597.75	- 1203.69				4
4	Cavaglio Torre	- 3779.68	+ 2390.44				5
5	San Giuseppe di Sizzano	- 86.46	+ 4852.40				6
6	Altipiano Bolei	+ 2754.65	+ 3026.88				7

Figure 6 Casini Soldner Coordinates of cadastral trigonometric vertices - *Source: AdT Novara*

The CAT program, realized in Matlab ® language, works by digitalization of paper archives similar to the ones reported in Figure 6. In order to start the procedure, it is necessary, in fact: to automatically recognize the common points in the database of the coordinates; to put into the correct order the points common to more than one origin and finally to write the system of equations as in the example of Table 1

An important preliminary work, therefore, led to the construction of a database of coordinates in digital format, “rediscovered” in the old cadastral archives. The work consisted also in searching for and retrieving all the network points used by the cadastre, coinciding with the network point of the IGM trigonometric net. In the Province of Novara there have been used:

- 310 tie points common to more than one origin, for a total of 798 tie combinations.
- cadastral vertices coincident with IGM trigonometric vertices (Roma 40).
- unknown cadastral origins.

The total number of points therefore amounts to 852 and generates a system of 1704 equations. They make it possible to estimate the coordinates of 310 points of the cadastral network which, in this way, rebuilds the connection among the various origins.

To these unknowns must be added those of the four transformation parameters for each of the 63 origins for a total of 872 unknowns. The system has, therefore, a good relative redundancy.

5 | ANALYSIS OF RESULTS

The compensation of this big block permits us us make some considerations to validate the accuracy and precision of the methodology, and at the same time, also the accuracy of cadastral points that have allowed the framing and construction of the “original plant maps”.

Regarding the obtained rotation and scale parameters, only the first ones showed values statistically different from zero, with standard deviation of a few *mgon*. The values of the scale factor are not statistically significant: Their application is, however, necessary to shape locally the local distortions of networks, even if they do not have a precise physical meaning.

Regarding the Figure 7 we may understand the precision with which the Gauss coordinates of the origins of the cadastral systems were estimated. The planimetric standard deviation estimated in compensation is shown on the ordinate, according to increasing values for each of the 62 origins of the province of Novara. It is less than 37 cm in 95% of the cases.

The compensation also allows us to jointly estimate the Gauss coordinates of the 310 points of the cadastral network common to two or more origins and their planimetric precision.

In Figure 8 these values are reported/displayed on the ordinate, ordered on the abscissa in crescent form. The possible considerations are coherent with what has already been observed about the accuracy of the origins of the cadastral systems: the precision is better than 41 cm in 95% of cases.

If we think that most of the maps were made on a scale of 1: 2000 (the graphicism error is about 40 cm) and that the precision of the detailed survey suffers just a very slight decrease, compared to that of the control network, these values therefore seem adequate, even today, for technical uses at this map scale.

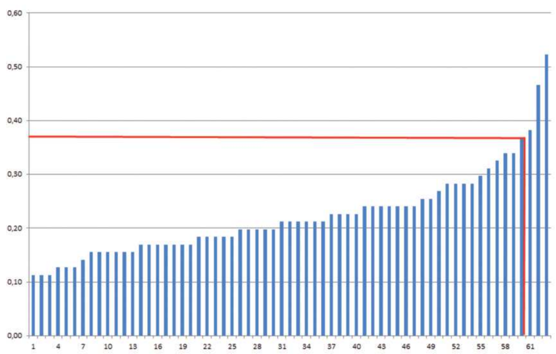


Figure 7 Planimetric standard deviation (m) of the 62 cadastral origins (<37 cm at 95%)

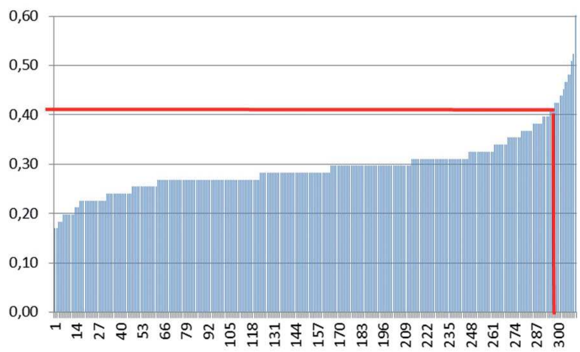


Figure 8 Planimetric standard deviation (m) of the 310 points of cadastral network (<41 cm al 95%)

It should be remembered that the formation of the cadastral trigonometric networks and IGM networks have been complex and, in part, simultaneous operations. The Cadastre used coordinates of the provisional type IGM vertices in northern Italy, in order not to wait for a network compensation that would have lengthened the cadastral operations.

The final coordinates could, therefore, be different from the ones utilized during the adjustment of cadastral networks. The coherence among cadastral networks and that of the IGM can be highlighted by the analysis of the standard deviations “*v*” (formula 12, visible in Figure 9 on 54 cadastral vertices, which are also IGM vertices. Although these deviations are less than 78 cm in 95% of the cases, the coherence between the 310 tie points common to adjacent cadastral networks is very good, with values less than 41 cm in 95% of cases (Figure 10).

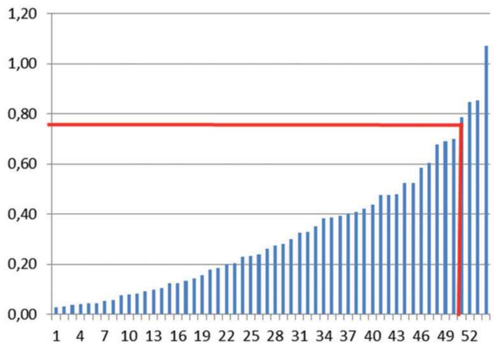


Figure 9 Crescent planimetric deviations [m] on 54 points of support (<78 cm at 95%)

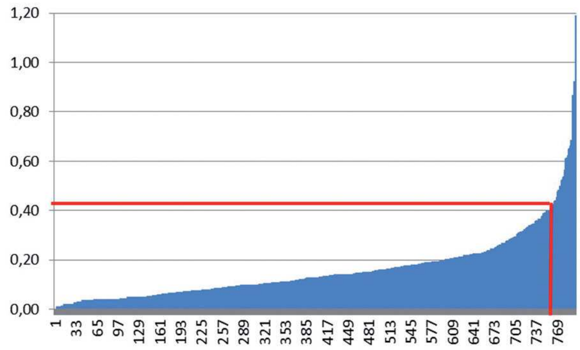


Figure 10 Crescent planimetric deviations [m] on 310 tie points common to a number of origins (<41cm at 95%)

The residual on the network control points appears to be particularly good, considering that the network examined lies on an area of about 50 km of development both in the East and in the North and that the network was detected and calculated more than a century ago.

The results obtained should be considered very good also in light of the tools and methods used by our old surveyors. This is to the merit and honor of this generation of surveyors who were able to achieve as much as they could with the tools and techniques of surveying available to them.

The result also demonstrates how this network is still able to support the cadastral cartography transformation, in the direction of the new reference systems, without a deterioration in precision and, as such, a transformation is compatible with the use of a single and more general reference system.

6 | CONCLUSIONS

Once the application of the CAT procedure to the Province of Novara had concluded, the same techniques were successfully applied to the various local systems of the entire cadastral network of the Piedmont Region.

The procedures reported here led ultimately to the transformation of the entire cadastral cartography of the entire Piedmont Region, first in the Roma 40 system, and then/eventually in the ETRF2000 reference system.

The latter work involved over 30,000 large-scale maps (mainly in 1:2000 scale map) and about 25,000 km^2 of surface area. Since there are not double coordinates in sufficient number for each cadastral origin, in the Provinces of Alessandria and Novara the CAT method was applied. In other cases the CS2UTM method was applied. The case of Novara alone was analyzed here, as it is more significant in terms of size, number of data and complex typology of reference systems.

The sheer scale of the task and the excellent results obtained have shown how by exploiting the historical geodetic data of the old cadastral networks, the monographs, the graphic diagrams of the networks, the registers, etc. the transformation between the reference systems can be performed in a rigorous fashion. All this, despite no prior information being available on the cadastral origins, but only by taking advantage of points common to networks of adjacent origins and a limited number of trigonometric points, known in the arrival system.

More than a century after its establishment, observing the accuracy and precision with which the original surveys have been conducted may surprise us. Indeed, testified to, after so many years, by the values of the discards and the coordinates of the tie points and the trigonometric vertices, even today they are in line with what is required by a modern 1:2000 scale cartography.

The rediscovered and valorized geodetic material was crucial because it allowed the reconstruction of the reference systems and their transformation. The transformation algorithms were then applied to the individual cadastral maps, for each origin, starting with the parameters estimated for each of them.

Equally astonishing was the verification of how, at the end of this operation, remarkably precise were the original historical “plant maps”. This, both in relation to the coherence with the neighboring origins and with the overlap with the technical paper or the recent orthophotocarta.

These papers, more recently found in the new reference systems and with much more sophisticated methods and tools, prove both the correctness of the procedure and the good work also performed by the surveyors of the “plant maps” which drew on the numerous local systems.

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References

Abidin H. Z., Haroen T.S., Adiyanto F. H., Andreas H., Gumilar I., Mudita I., Soemarto I. (2015).

On the establishment and implementation of GPS CORS cadastral surveying and mapping in Indonesia. Survey Review 47 (340), 61-70. DOI: http://dx.doi.org/10.1179/1752270614Y.0000000094

Bildirici, I. O. (2003). Numerical inverse transformation for map projections. *Computers & Geosciences* 29: 1003-1011.

Bin A. Y., Chai G., P. (1996). *Improving cadastral survey controls using GPS surveying in Singapore*. Survey Review 33 (261) 488 495

Boaga, G.(1953). Sulla rappresentazione cilindrica congruente di Soldner e sui problemi geo-cartografici dell'ingegneria. Rivista del Catasto e dei Servizi Tecnici Erariali, n. 5, 1953. pagg. 285-296.

Bonifacino, B. (1968). La rappresentazione di Gauss nella formazione delle nuove mappe catastali. *Rivista del Catasto e dei Servizi Tecnici Erariali,* n. 5-6, 1968. pagg. 217-240.

Cina A. (2008). La carta catastale in un sistema globale. In: *Rivista dell'Agenzia del Territorio*, vol. 1, pp. 13-26. - ISSN 1593-2192

Cina A., Ferrante F.C., Piras M., Porporato M.C. (2012). La trasformazione dal *datum* catastale ai *datum* Roma 1940 e ETRF2000. In: *Territorio Italia*, vol. 01/12, pp. 107-119. - ISSN 2240-7707

Cina, A.; Manzino, A.M.; Manzino, G. (2015). Recovery of cadastral boundaries with GNSS equipment. In: Survey Review. - ISSN 0039-6265

Di Filippo, S. (1995). Sulla trasformazione delle coordinate plano-cartografiche dalla rappresentazione di Cassini-Soldner alla rappresentazione di Gauss-Boaga e viceversa (parte 1°). *Rivista del Dipartimento del Territorio*, n. 3 1995. pagg. 151-186.

Di Filippo, S. (1996). Sulla trasformazione delle coordinate plano-cartografiche dalla rappresentazione di Cassini-Soldner alla rappresentazione di Gauss-Boaga e viceversa (parte 2°). *Rivista del Dipartimento del Territorio*, n. 1 1996. pagg. 105-187.

Doytsher, Y., Hall, J.K., (1997). Gridded affine transformation and rubber-sheeting algorithm with FORTRAN program for calibrating scanned hydrographic survey maps. *Computers & Geoscience* 23, 785-791.

Felus Y., A., (2007). On the positional enhancement of digital cadastral maps. *Survey Review* 39 (306), 268-281. DOI: http://dx.doi.org/10.1179/175227007X197183

Guiducci, A. (1953). Trasformazione delle coordinate Cassini-Soldner di piccoli sviluppi catastali in coordinate Gauss-Boaga. *Rivista del Catasto e dei Servizi Tecnici Erariali*, n. 2, 1953. pagg. 109-113.

Inghilleri, G. (1974): *Topografia Generale*. Torino, Utet, 1974.

Kao S-P., Tu C-P., Chen C-N. (2013). A coordinate vector correction method to improve the traditional affine transformation graphic digitized cadastral map. 2013. *Survey Review* 45 (332) 352-356. DOI: http://dx.doi.org/10.1179/1752270612Y.0000000032

Kraus, K. (1994). *Fotogrammetria*. Vol.1 – Teoria e applicazioni. Traduzione e ampliamento di Sergio Dequal. Torino, Levrotto & Bella, 1994.

Manzino, A. (2017). *Quaderni di Topografia* Vol.1 – Torino, Levrotto e Bella

Manzino, A.M., Dabove, P. (2013). *Quality control of the NRTK positioning with mass-market receivers.* In: Hsues H (Ed.) Global Positioning Systems: Signal Structure, Applications and Sources of Error and Biases, Nova Science Publisher Inc., pp. 17-40.

Pàroli, A. (1943). Il problema della trasformazione delle coordinate nella rappresentazione conforme di Gauss e l'unificazione delle reti trigonometriche del nuovo stato. *Rivista del Catasto e dei Servizi Tecnici Erariali*, n. 3, 1943. pagg. 213-227.

Pàroli, A. (1948). *Triangolazioni topografiche e del Catasto*. Milano, Hoepli, 1948.

R. Commissione Geodetica Italiana: *Elementi della rete geodetica fondamentale a nord del parallelo di Roma*. 1908.

R. Commissione Geodetica Italiana: *Elementi della rete geodetica fondamentale a sud del parallelo di Roma*. 1919.

Stoppini, A. (1992). Procedimento per la trasformazione diretta e inversa delle coordinate piane dal sistema catastale al sistema nazionale. *Rivista del Catasto e dei Servizi Tecnici Erariali*, n.1, 1992. pagg. 13-36.

Surace, L. (1991). L'integrazione tra sistemi cartografici locali e sistema cartografico nazionale per la geo-referenziazione delle informazioni territoriali. *Bollettino della Associazione Italiana di Cartografia*. 81-82/1991

Surace, L. (2004). La georeferenziazione delle informazioni territoriali. "*Bollettino di Geodesia e Scienze Affini*", vol. III, n.6, pp.168-179.

Targa, G. Vettore, A. (1994). Nota tecnica sulle trasformazioni di coordinate plano-cartografiche dal sistema catastale Cassini-Soldner al sistema nazionale Gauss-Boaga. *Rivista del Dipartimento del Territorio*, n. 3 1994. pagg. 109-124.