*Alberto Cina
**Flavio C. Ferrante
*Marco Piras
***Chiara Porporato

# Transformation from the Cadastral DATUM to the Roma 1940 and ETRF2000 DATUM

**Keywords:** **Cassini-Soldner, cadastral map, ETRF2000, DATUM transformation.**

**Abstract**

This paper shows a geodetic procedure of transformation from the cadastral DATUM to the Roma 1940 and ETRF2000 DATUM as developed at the Politecnico di Torino – DIATI. It has been implemented in the Piedmont Region, which presented a very complex situation for the several (about 540) small extension axis systems. This was done after an experiment was carried out in a collaboration between Agenzia del Territorio and Politecnico di Torino – DIATI, with respect to an agreement between Politecnico di Torino, Regione Piemonte, and CSI Piemonte.

## INTRODUCTION

The cadastral map is an indispensable tool for land management. It is the only large-scale map that covers the entire Italian territory and represents possession. The Agenzia del Territorio has vector maps in the native reference system, and the "original plant" maps are currently in raster format. The interoperability of cartographic databases of other agencies devoted to land management is important to correctly implement any programming operation, planning, and land management. From here, the need for sharing the same reference system or DATUM thus arises.

The cadastral reference system is historically based on Bessel ellipsoid (Bessel 1841 DATUM), which assumed various local orientations. The cartographic representation adopted by that DATUM, except in limited areas, was the Cassini-Soldner representation, with more than 800 small and large extension origins (axis systems). The adoption of the Roma 1940 reference system, which was developed by a first order geodetic network of IGM (Istituto Geografico Militare) with Gauss-Boaga cartographic coordinates, thus poses a cadastral datum transformation problem (Paroli, 1943).

It must be considered that the evolution of reference systems in the international ambit allows for the transposition of the European spatial data sharing directive INSPIRE (Infrastructure for Spatial Information in Europe) and thus the adoption of the new WGS84 (World Geodetic System 1984) DATUM in Italy; this is consistent with a system where measurements and calculations are made with GPS (Global Positioning System) or GNSS (Global Navigation Satellite systems). At the WGS84, abbreviation was found to better associate its geodetic construction (frame), making the system a usable reference. In Italy, the frame is constructed by the National Dynamics Network (RDN) and realised by

*Politecnico di Torino – Department of Environment, Land and Infrastructure Engineering
** Agenzia del Territorio – Central Directorate Cadastre and Cartography
*** Politecnico di Torino – Department of Architecture and Design

IGM, giving the ETRF2000 frame (Baroni *et al.*, 2012). Therefore, we will give the name UTM-ETRF2000 to the cartographic coordinates resulting from this DATUM, in the Gaussian representation.
It should also be recalled that the exchange between Roma 1940 and ETRF2000 DATUM is solved by the procedures of continuous transformation, as well as identified and put into practice by IGM in the VERTO approach (Donatelli *et al.*., 2002). This approach is based on the interpolation of grid variation in geographic coordinates among DATUM, as represented in a file with the extension ".GK2." Therefore, it is sufficient to transform the cadastral DATUM into one of the two DATUM (e.g. Roma 1940) and subsequently apply the VERTO procedure to carry out the further changes in the reference system (e.g. ETRF2000).
Consequently, the geodetic procedure for the transformation of the cadastral DATUM, applicable to both the Roma 1940 and the ETRF2000 DATUM as developed at the Politecnico di Torino – DIATI (Land, Environment and Infrastructure Engineering Department), can be exposed. This has been implemented in the Piedmont Region for several (540) small extension axis systems, after experimentation in accordance with the Agenzia del Territorio and Politecnico di Torino – DIATI and the agreement between Politecnico di Torino, Regione Piemonte, and CSI (Consorzio per il Sistema Informativo) Piemonte.

## THE CADASTRAL DATUM TRANSFORMATION

In general, the transformation between DATUM is performed by comparing measurements and known coordinates in the two systems and estimating the transformation parameters according to a given model (generally a rototranslation with a scale variation). It is only possible to compare the coordinates of points in the two reference systems if the coordinate systems are consistent with each other (e.g. both cartographic systems or geographic systems).
In this case, the Bessel 1841 cadastral DATUM with Cassini-Soldner cartographic coordinates must be transformed to one of two systems: Roma 1940 with Gauss-Boaga cartographic coordinates and ETRF2000 with UTM-ETRF2000 coordinates. The calculation procedures for such transformations may be different, even if they lead to the same goal. One can start with cartographic or geographic coordinates, based on the knowledge of double points of coordinates or that of cadastral origins, even in the single system of arrival. In summary, two procedures are possible:

- Method for **double points of coordinates**: This allows for the transformation estimation of the cadastral DATUM to the Roma 1940 DATUM (Di Filippo, 1995). by using points with double coordinates known in the two systems, which are not necessarily the origin. In general, consistency is assumed in maps within the same origin in order for the search of the double points to be carried out from the origins and not from the maps (Di Filippo, 2003).
- Method for **cadastral origins**: This methodology is based on geodetic and cartographic considerations that, from the knowledge of the cadastral origin in the arrival system, allow for the re-projection of the cadastral points to ETRF2000 or Roma 1940 DATUM, passing through the surfaces of ellipsoids.

Within its own information system, the Agenzia del Territorio uses the above procedure for double points. The procedure for cadastral origins will be addressed in this paper. However, it should be noted that the two methods are not strictly mutually exclusive but, on the contrary, can be integrated, as described below. The knowledge of cadastral origins allows for the transformation to perform even in the absence of a sufficient number of points of double coordinates, a situation that often appears with small origins. Within the procedure for cadastral origins, it is also possible to analytically generate points of double coordinates for DATUM transformations with the method for double points; this

is used by the Agenzia del Territorio.
On the other hand, having points of double coordinates allows for the re-determination of the cadastral system origin that generated them, as well as the estimation of the coordinates in the final system and its variance-covariance matrix by the principle of least squares. This gives an estimate of the accuracy and reliability of the origin or a validation of the data present in the monograph. The method can then be adapted to determine origins that have lost their sources or documentation, or resolve origins that have been placed on an "ideal point", i.e., without materialisation Both of the above approaches can be applied depending on the type of geodetic material available. In the next section, the procedure will be summarised for geodetic cadastral origins (for more details, see Cina (2008)).

## GEODETIC PROCEDURE FOR CADASTRAL ORIGINS

This procedure is based on the consideration that for the initial and arrival DATUM, the geodetic azimuth $\alpha$ and the geodetic length s remain virtually unchanged while the shape of the ellipsoid is changed. These values, (s, $\alpha$), in polar geodetic coordinates, can be found starting with the Cassini-Soldner values, (Y, X), and using the formulas of Soldner (1) (Inghilleri, 1970) through the calculation of the spherical excess $3\varepsilon$:

$$s = \sqrt{(Y + \varepsilon X)^2 + (X - 2\varepsilon Y)^2} \qquad \alpha = \arctan\frac{(Y + \varepsilon X)}{(X - 2\varepsilon Y)} \qquad 3\varepsilon = \frac{XY}{2\rho_0 N_0} \tag{1}$$

where $\rho_0$ and $N_0$ are associated with the principal radii of curvature.

We consider the transformation of cadastral DATUM to be a two-dimensional problem, and use the coordinates in the Gaussian plane. In this cartographic plane, we can use lengths and azimuth of the geodetics transforms for the transport of the coordinates (2):

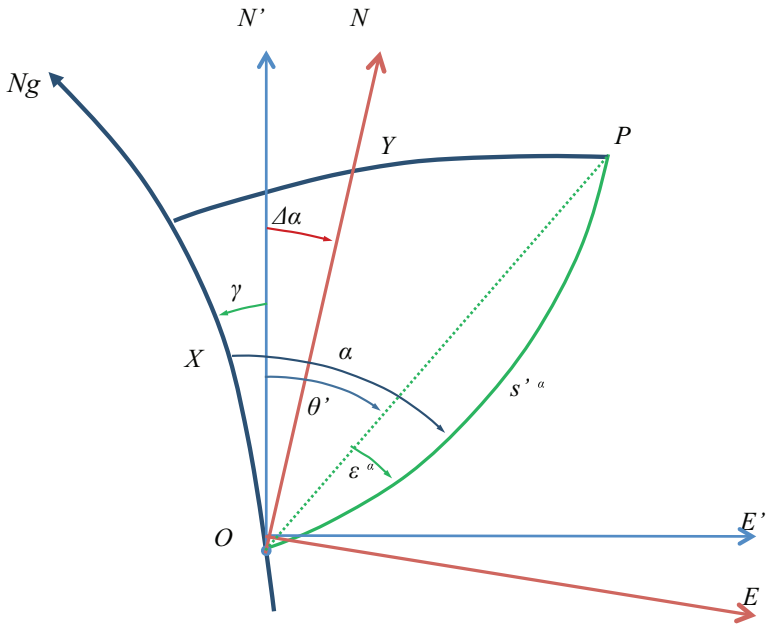
$$\begin{aligned} E' &= E_0 + s' \sin\theta' \\ N' &= N_0 + s' \cos\theta' \end{aligned} \tag{2}$$

Here, s' and $\theta$' are, respectively, the length of the geodetic transform and the angle of the chord that subtends it in the Gaussian plane. Assuming the coordinates $E_0$, $N_0$ of the origin in the arrival system, the coordinates E', N' are obtained in an "intermediate" Gaussian plane, rotated and scaled with respect to the arrival DATUM (Figure 1). It is only necessary, therefore, to estimate the rotation parameters $\Delta\alpha$ and scale factor $\lambda$ from a few points of double coordinates. This estimate can be made for each origin or group of origins. From the Gaussian "intermediate" plane, we can arrive at the final system with coordinates E, N, applying a rotation and a scale factor as follows (3):

$$\begin{pmatrix} E \\ N \end{pmatrix} = \lambda \begin{bmatrix} \cos\Delta\alpha & \sin\Delta\alpha \\ -\sin\Delta\alpha & \cos\Delta\alpha \end{bmatrix} \begin{pmatrix} E' \\ N' \end{pmatrix} \tag{3}$$

The rotation can be explained by the different orientation of the ellipsoid, while the scale factor allows one to take into account the inevitable distortions in the geodetic networks that materialise different frames.

**Figure 1** Reference and coordinate systems: Y, X – Cassini-Soldner
E', N' – Gaussian coordinates in an intermediate system, and E, N – Gaussian coordinates in the arrival system.

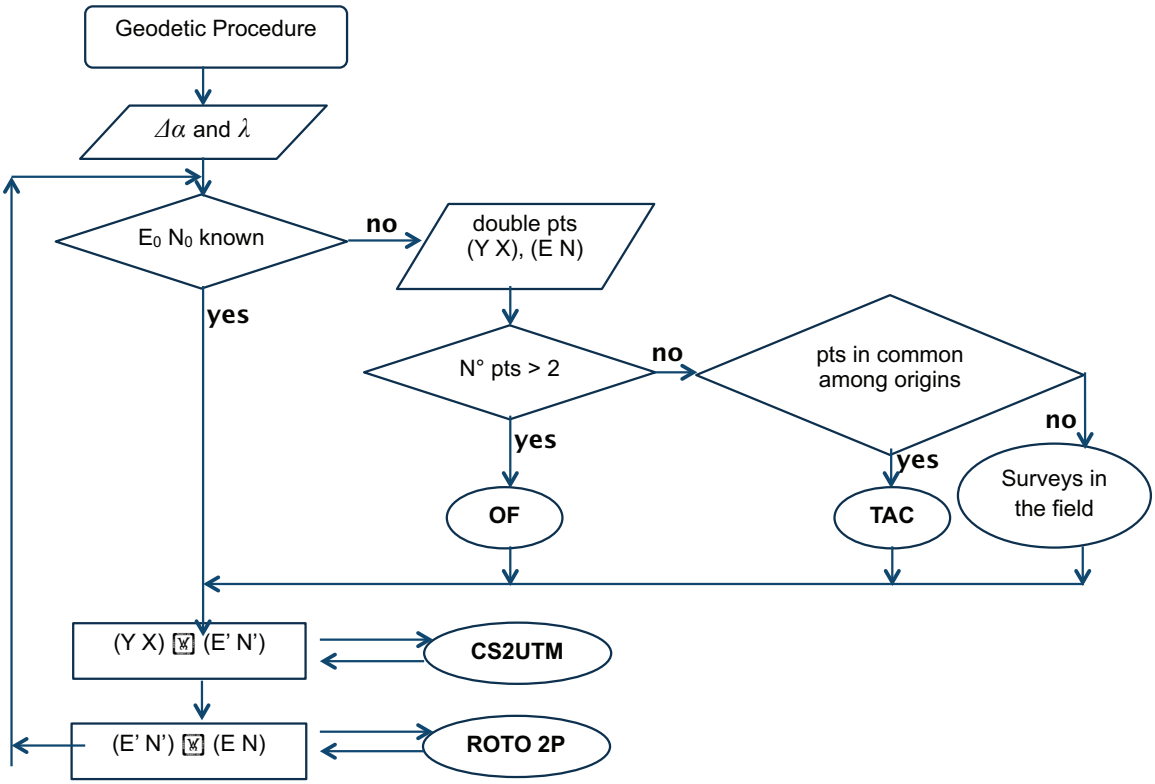



The calculation of s' and θ' can be carried out starting from the geodetic polar coordinates (s, α), taking into account the angular deformation of the Cassini-Soldner ($\delta_{CS}$) representation, the Cassini-Soldner ($m_{CS}$) and Gaussian ($m_{Gauss}$) coefficients of linear deformation, and the elements of cartographic reduction, such as the meridian convergence (γ) and the angular correction to the chord (ε) (4). Therefore, it is necessary to project (s, α) from the Cassini-Soldner plane to the surface of the ellipsoid, and then to the Gaussian plane.

$$\theta' = \alpha + \delta_{OP} + \gamma_0 - \varepsilon_{OP} \qquad s' = s \frac{m_{Gauss}}{m_{CS}} \tag{4}$$

The transformation can be performed with a specified geodetic procedure, such as the CS2UTM procedure in Figure 2. Here, the knowledge of the cadastral origin in the arrival system, and its eventual estimation if it is unknown or uncertain, is clearly important. The methods of determining the cadastral origin, known as OF (Fictitious Origin) and TAC (Cadastral Aerial Triangulation), will be presented below. In the cases where there are grounds for their application, an alternative solution is given by determining the coordinates in the country.

**Figure 2** Transformation from the cadastral DATUM in the Cassini-Soldner coordinates (Y, X) to the arrival DATUM in Gaussian coordinates (E, N).

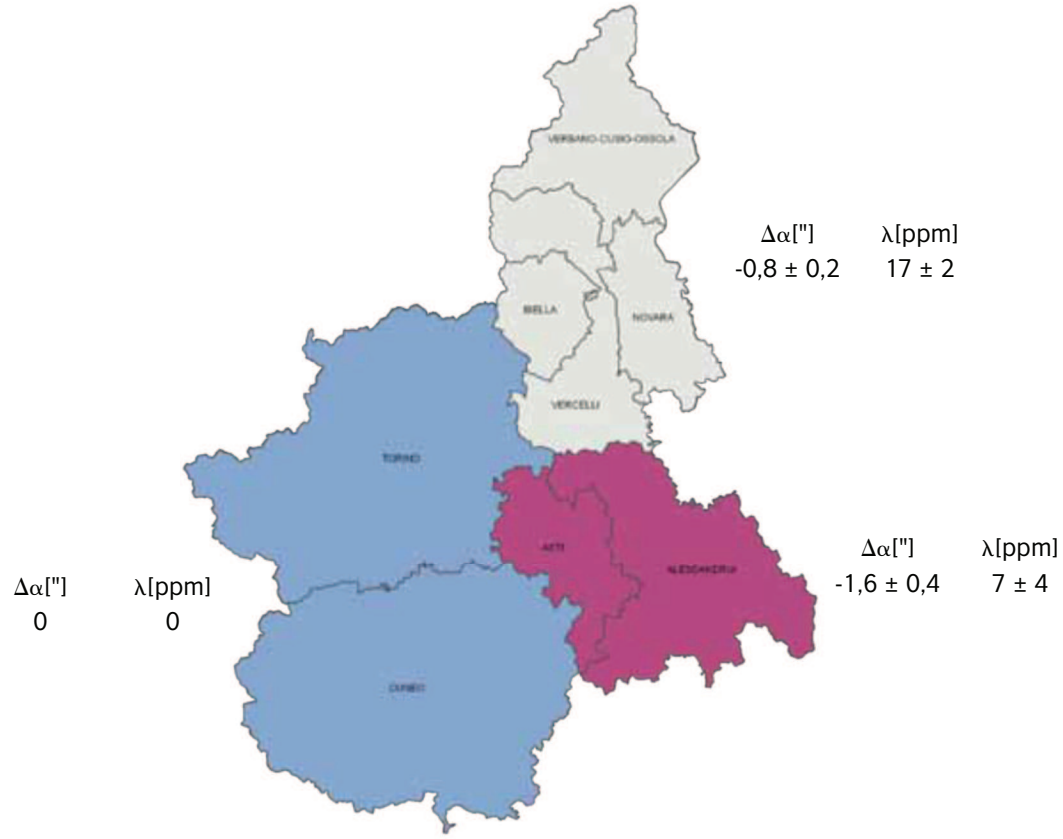



## ESTIMATION OF ROTATION AND SCALE PARAMETERS: THE REGINE PIEMONTE CASE

As mentioned above, this estimation involves the transformation of the Cassini-Soldner coordinates into an intermediate system on the Gaussian plane, which are rotated and scaled with respect to the arrival system. The estimation of these parameters does not generally require many points with double coordinates if they are of good quality and well represent the reference systems. Involving as many points as possible in the estimation, however, allows for the evaluation of the quality of the coordinates in the two reference systems, through the use of least squares residual tests (Cina, 2002). This estimation can be made for each cadastral origin or for groups of origins with similar parameters. For the special case of the Regione Piemonte, the parameter values estimated for each province were significantly different. Consequently, an estimate for the whole region provided high mean square deviations. The various attempts at the clustering of the provinces have led to the identification of three main areas that are consistent with regards to the orientation parameters.

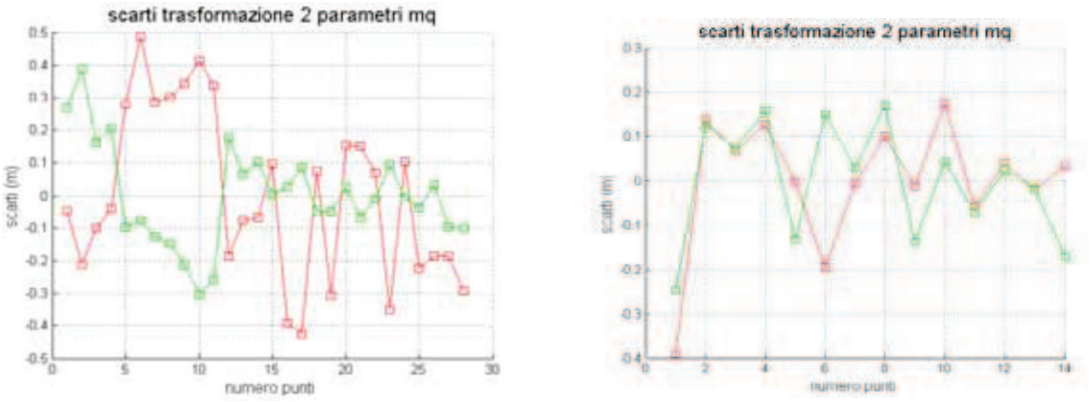
These three different clusters, shown in different colours in Figure 3, respectively involve three different geographical areas in the Piemonte: South-West (provinces of Torino and Cuneo), East (provinces of Alessandria and Asti), and North (provinces of Vercelli, Verbano Cusio Ossola, Biella, and Novara). The parameters are summarised in Figure 3 and can be used for DATUM transformation. The transformation between cadastral DATUM and ETRF2000 can also be solved employing these parameters.

**Figure 3** Estimated orientation parameters for each Piedmont cluster.




The residuals regarding the points with double coordinates for two of the three clusters are shown in Figure 4. The residuals are always lower than 50 cm, even when there are large distances between the vertices.

**Figure 4** Estimated least squares residuals for the points used to estimate the transformation parameters. Green is East and red is North.



## DETERMINATION OF THE CADASTRAL ORIGIN

In the transformation described above, the reliability of the coordinates of the cadastral origin is very important. These values have been reported both in Roma 1940 and Cassini-Soldner (monographs are conserved in the archives of the Agenzia del Territorio, Figure 5) or only in Cassini-Soldner (Figure 6).

**Figure 5** – Monograph of a cadastral vertex with coordinates in Cassini-Soldner and Gauss Boaga.

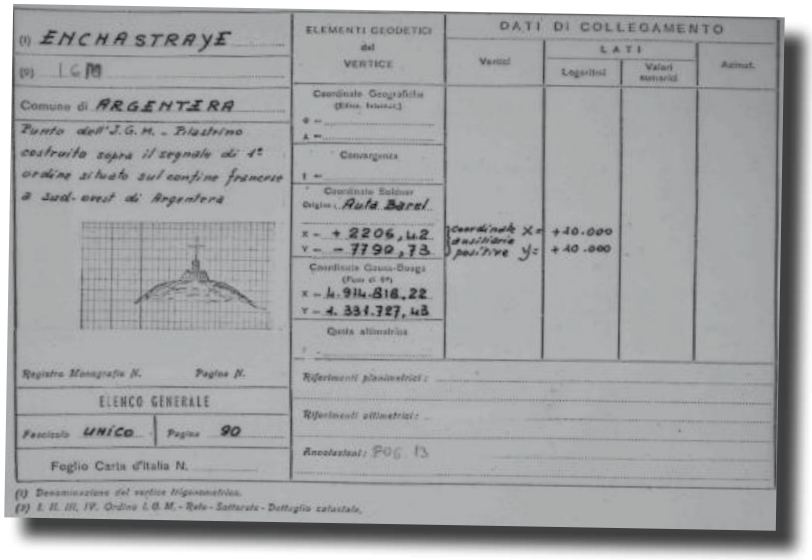
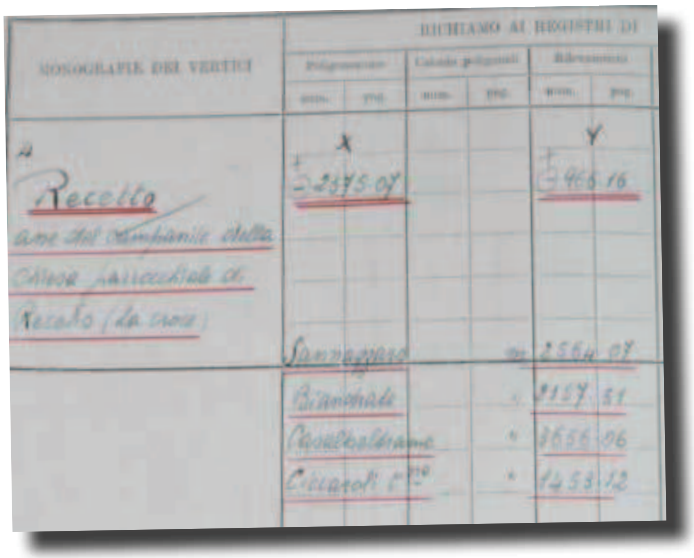



**Figure 6** – Coordinates of the vertex only reported in Cassini-Soldner.




If the coordinates of the vertex are not available in the arrival reference system, it is possible to

determine them from direct ground measurements. Examples of surveys are shown in Figure 7a and b, where the cadastral origin, as defined as the axis of the tower, has been determined both in Roma 1940 and ETRF2000.
Starting from a GPS baseline, a forward intersection can then be realised.

**Figure 7** Extremes of the GPS baseline for the determination of the coordinates of the cadastral origin, using a forward intersection.

a b

There are some alternative approaches, if points with double coordinates close to the cadastral origin are available or common points between neighbouring origins are existent.
These alternative approaches are known as “Fictitious Origin (OF)” and “Cadastral Aerial Triangulation (TAC)”.

**Fictitious Origin (OF) method:** coordinates of the origin for each cadastral axis system are estimated, starting from the points with double coordinates and using an iterative geodetic method (Bendea *et al.*., 2009). Here, an approximate position of the emanation centre is required. The calculation is rapidly convergent (two to three iterations) and it gives the least squares estimation of the origin coordinates in the arrival system, with the related variance-covariance matrix. This method allows the data-snooping operations, which are useful in order to identify eventual vertices with no coherent coordinates in both reference systems.
A particular case, reported below, is the coordinate determination of the “ideal” or “Ω” point, which is an origin with a large extension in the North-East section of the Piemonte. Only coordinates in the Bessel DATUM are available in the archives (Figure 9), but several double points (Cassini-Soldner and Gauss Boaga) are also available. The coordinates in Roma 1940 and the related standard deviation (SD) have been estimated, applying the method of the “Fictitious Origin:”

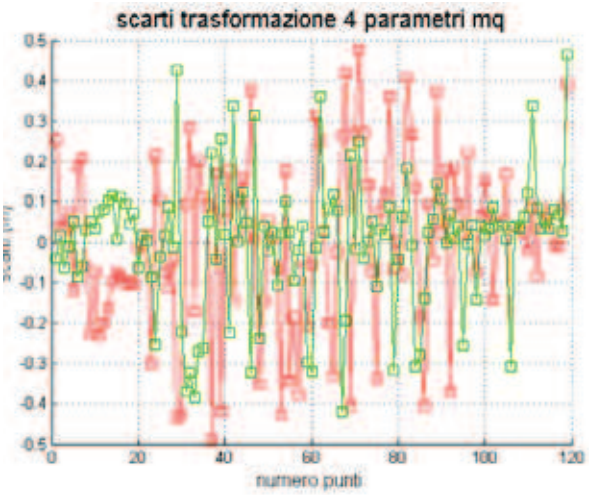
| | East Gauss Boaga | North Gauss Boaga |
|---|---|---|
| point Ω | 1437863.55 m ± 0.02 m | 5033310.73 m ± 0.02 m |

**Figure 8** Ideal Point "Ω" of Vercelli in Salussola Town. **Figure 9** Coordinates of the "Ω" point in Bessel DATUM.



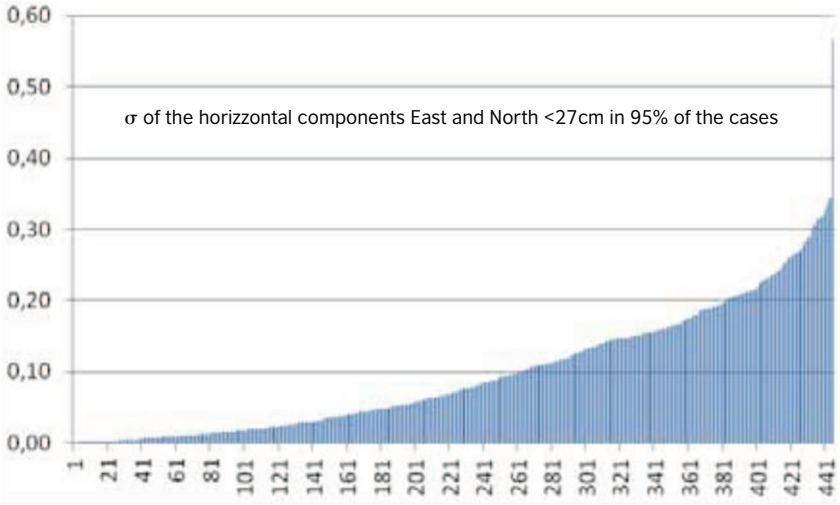
It is quite interesting to see that the coordinates of the ideal point Ω have been estimated to a precision of 2 cm, with least squares residuals (comparing double points) less than 50 cm (Figure 10). This result is quite incredible considering the spaciousness of the territory for the cadastral network of this origin. In this case, some distances between vertices and the origin are greater than 100 km.

**Figure 10** Least squares residuals for the double points used to estimate the ideal point "Ω." Here, East is green and North is red.



This procedure has been used to newly estimate the 445 cadastral origins in Piemonte. The precision of the coordinate estimation is summarised in Figure 11, where the horizontal residuals of the double points are less than 1 m.

**Figure 11** Horizontal standard deviations for 445 origins in the Piedmont, newly estimated with the Fictitious Origin method



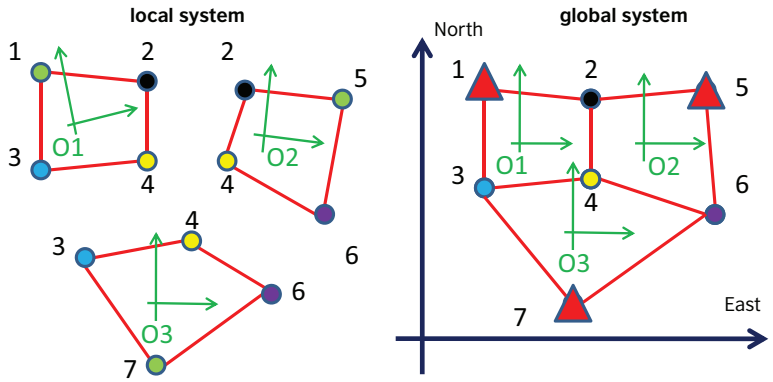

**Cadastral Aerial Triangulation (TAC) Method**

this method can be used in order to estimate the origin coordinates when there is not a sufficient number of points with double coordinates for each axis system.

Like photogrammetric aerial triangulation for "independent model" (Kraus, 1994), Cadastral Aerial Triangulation (TAC) is based on points that belong to several origins (tie points) and a few trigonometric vertices with double coordinates (control points), as seen in Figure 12.

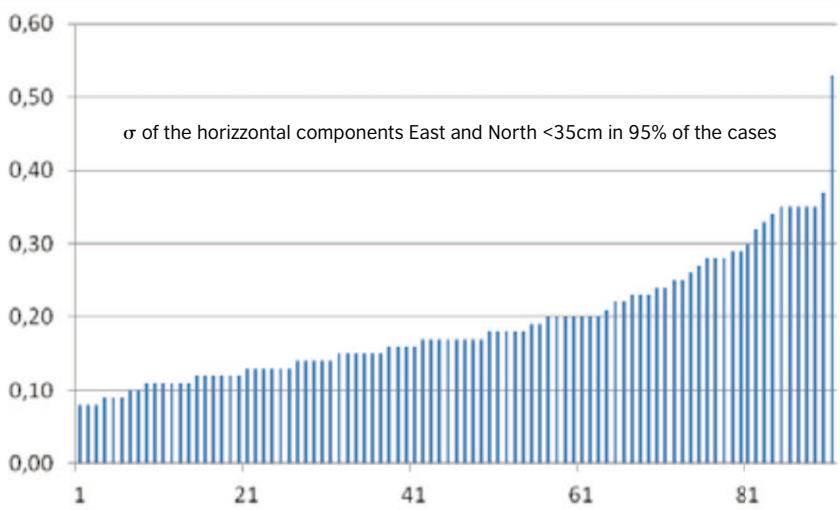
The block is adjusted and oriented in the arrival reference system, using the least squares procedure; here, the variance-covariance matrix of the parameters is also estimated (Cina *et al.*., 2011).

**Figure 12** TAC: In this example, three origins (O1, O2, O3) with tie points (2, 3, 4, 6) and three points with double coordinates (1, 5, 7) are recomposed in the global system (East and North).



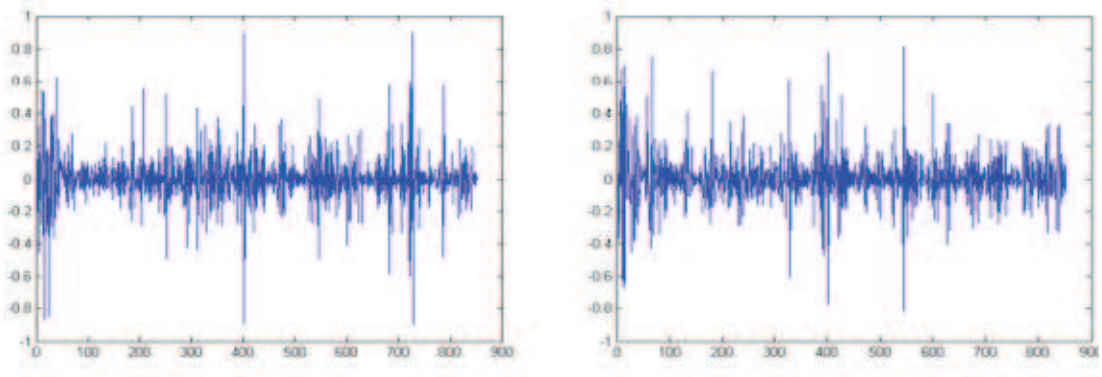
In the case of the Regione Piemonte, in some parts of the Alessandria and Novara provinces, monographs of the cadastral origins with double coordinates were not available. Only the coordinates of the vertices in Cassini-Solder were obtained.
A large number of points belonging to different origins and some cadastral vertices with coordinates defined in the Gauss Boaga system have been identified, in order to use the TAC. Ninety-one small origins were solved, starting with more than 850 tie points or common points. The horizontal SD of the estimated cadastral origins obtained with the bundle block adjustment is shown in Figure 13.

**Figure 13** Horizontal SD of the 91 origins in the Piedmont, estimated with the Cadastral Aerial Triangulation method.




The residuals of the least squares estimation, considering the control points, are always within 1 m, as shown in Figures 14 and 15.

**Figure 14** – North component residuals on tie points and ground control points.
**Figure 15** – East component residuals on tie points and ground control points.



## THE DEVELOPMENT OF TRANSFORMATION PARAMETERS BETWEEN CADASTRAL DATUM AND ROMA 1940 DATUM BY THE AGENZIA DEL TERRITORIO

As foreseen in the "Protocollo d'Intesa" between the Agenzia del Territorio and Regione Piemonte, the Politecnico di Torino has generated a homogenous grid of points, for each cadastral axis system and each map sheet, using the procedure described above. These points are known in double coordinates, both in the cadastral and ROMA 1940 systems, and are compatible with the procedures used by the Agenzia del Territorio.

The Agenzia del Territorio realised appropriate controls when their own cartographic database was considered; subsequently, it has enhanced these grids and allowed for, with their own standard procedure, the transformation of the vector maps of the Piemonte into Roma 1940 DATUM. When this transformation is complete, the cartographic files will be made available to the Regione Piemonte through the interchange system.

At the moment, this process has been completed for the following provinces: Asti, Biella, Verbania, and Vercelli. The activities for the remaining provinces will be completed by the end of the first semester of 2012.

It is important to highlight that the Istituto Geografico Militare provided the Agenzia del Territorio, according to recent agreements, with the grids of the entire national territory, allowing for the transformation from Roma 1940 DATUM to ETRF2000 DATUM. The cadastral maps in this last reference system will be made available soon with the interchange system.

## CONCLUSIONS

The geodetic procedure described above is an approach devoted to transform the cadastral DATUM to new forms such as Roma 1940 and ETRF2000. This procedure has been used in the Regione Piemonte and it has allowed for the determination of more than 540 cadastral origins without the use of ground surveys, excluding two limited and particular cases.

This approach has also been used to estimate the DATUM transformation parameters, identifying three homogenous zones where these values can be considered constant.

The analysis of the residuals obtained with the least squares method has enabled us to identify erroneous points with double coordinates and to evaluate the coherence between the coordinates considering both the arrival and departure DATUM.

The described procedure can be adopted in different cadastral areas and can be applied to both situations with a large extension origin (i.e., two cases in Piemonte) and to situations with ideal points, in particular where there is a large number of axis systems with small extensions. For instance, this is the case for some provinces of the Regione Veneto and in the provinces of Palermo, Potenza, and Matera. The Agenzia del Territorio has not yet elaborated on these cases.

**Acknowledgments**
The authors would like to thank Dr. Gianni Siletto of the Regione Piemonte, as well as Eng. Mauro Vasone and Drs. Marzio Pipino and Antonio Chiaffrino of the CSI Piemonte, for making the necessary documents available in order for the results to be presented.

**References**

Baroni, L., Cauli, F., Donatelli, D., Farolfi, G., and Maseroli, R. *La rete dinamica nazionale (RDN) ed il nuovo sistema di riferimento ETRF2000*. Disponibile in http://87.30.244.175/rdn/rdn_download/relazione.pdf. (visitato il 9/01/2012).

Bendea, H., Cina, A., De Agostino, M., Manzino, A., Piras, M., and Porporato, C. *La ricomposizione dei piccoli sistemi d'asse catastali nel sistema WGS84* in Atti della 13° Conferenza nazionale ASITA (2009).

Cina, A. *Trattamento delle misure topografiche. Torino: CELID.* 125 pp. ISBN 88-7661-534-2 (2002).

Cina, A. L*a carta catastale in un sistema globale in Rivista dell'agenzia del Territorio,* vol. 1, pp. 13–26, ISSN: 1593–2192 (2008).

Cina, A., Ferrante, F.C., Manzino, A.M., and Porporato, C. *Metodologie geodetiche e cartografiche per la ricomposizione della mappa catastale nel sistema UTM-ETRF2000. in Bollettino Della Società Italiana Di Fotogrammetria E Topografia*, vol. 1, pp. 27–40, ISSN: 1721-971X (2010).

Cina, A., Caione, S., Chiaffrino, A., Ferrante, F.C., Garretti, L., Manzino, A.M. Pipino, M. Piras, M. Porporato, C., and Siletto, G.B. *La trasformazione di mappe catastali in Regione Piemonte, nel sistema WGS84 in Bollettino Della Società Italiana Di Fotogrammetria E Topografia,* vol. 1, pp. 27–40, ISSN: 1721-971X, (2012) in stampa.

Di Filippo, S. *Sulla trasformazione delle coordinate plano-cartografiche dalla rappresentazione di Cassini-Soldner alla rappresentazione di Gauss-Boaga e viceversa. In Rivista del Dipartimento del Territorio*, vol. 3, pp. 151–186 (1995).

Di Filippo S. S*ul passaggio delle coordinate plano-cartografiche al sistema WGS84 e viceversa in Rivista dell'Agenzia del Territorio*, vol. 1, pp. 85–90 (2003).

Donatelli, D., Maseroli, R., and Pierozzi, M. *La trasformazione tra i sistemi di riferimento utilizzati in Italia in Bollettino di Geodesia e Scienze affini*, vol. 4, pp. 247–281 (2002).

Inghilleri, G. *Topografia Generale.* Torino: UTET (1970).

Kraus, K. *Fotogrammetria. vol.1 – Teoria e applicazion*i. Traduzione di Sergio Dequal. Torino: Levrotto & Bella. 518 pp (1994).

Pàroli, A. *Il problema della trasformazione delle coordinate nella rappresentazione conforme di Gauss e l'unificazione delle reti trigonometriche del nuovo stato in Rivista del Catasto e dei Servizi Tecnici Erariali*, vol. 3, p